

Performance Comparison of Subgrid Techniques in Acoustic Simulation Using the Type-M and Type-C Constrained Interpolation Profile Methods

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Received November 19, 2011; accepted March 8, 2012; published online July 20, 2012

The constrained interpolation profile (CIP) method, a type of method of characteristics (MOC), is a novel low-dispersive numerical scheme. In an earlier study, we applied the CIP method to quantitative analyses of sound wave propagation. However, a new grid system is important for the CIP simulation of complicated heterogeneous media or large-scale simulations of wave propagation. In this study, we examined a subgrid technique for use with acoustic wave simulation using the type-M and type-C CIP methods. The results indicate that this technique for the CIP methods has advantages of small memory requirements and faster calculation. © 2012 The Japan Society of Applied Physics

To date, as a result of computer development, numerical analysis for sound wave propagation in a time domain has been investigated widely.¹⁾ For acoustic and ultrasound imaging, the development of accurate numerical schemes is important.

The constrained interpolation profile (CIP) method, a novel low-dispersive numerical scheme²⁻⁸⁾ is a type of method of characteristics (MOC).⁹⁾ We used CIP method for numerical analyses of sound wave propagation in an earlier study.⁶⁻⁸⁾

However, CIP methods for new grid systems are required for the numerical simulation of complicated heterogeneous media or large-scale simulations of wave propagation. To overcome this problem, subgrid techniques are proposed for other simulation methods of wave propagation.¹⁰⁾ This technique for the type-C CIP method has been partially introduced.¹¹⁾ As for the type-M CIP method,^{4,8)} however, it has not been investigated. The type-M CIP method is a simple technique with smaller memory use and less calculation time required than the type-C CIP method.⁸⁾ Therefore, from the point of reduction in the calculation cost, a subgrid technique for the type-M CIP method is also important. Subgrids are defined as those smaller than the surrounding grids: we can use suitable multisize grids in an analysis domain according to a subgrid technique for the CIP-MOC simulation of sound wave propagation.

In CIP analysis, the governing equations for linear acoustic fields (a lossless medium) are transformed into advection forms. For example, for the calculation of x -direction advection, the advection equation is given⁶⁻⁸⁾ as

$$\frac{\partial(p \pm Zv_x)}{\partial t} \pm c \frac{\partial(p \pm Zv_x)}{\partial x} = 0. \quad (1)$$

In this equation, p is the sound pressure, v_x is the particle velocity, Z signifies the characteristic impedance (i.e., $Z = \sqrt{\rho K}$), and c represents the sound velocity in the medium used (i.e., $c = \sqrt{K/\rho}$). Here, ρ denotes the density of the medium, and K represents the bulk modulus.

In addition, through the simple spatial differentiation of the equations, the equations of the derivatives are given as

$$\frac{\partial(\partial_x p \pm Z \partial_x v_x)}{\partial t} \pm c \frac{\partial(\partial_x p \pm Z \partial_x v_x)}{\partial x} = 0 \quad (2)$$

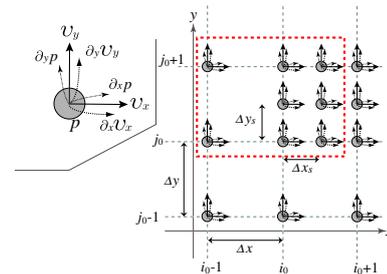


Fig. 1. (Color online) Subgrid technique for the type-M CIP method.

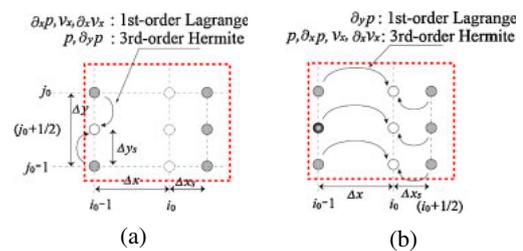


Fig. 2. (Color online) Treatment of the boundary. (a) first step and (b) second step.

We can calculate sound wave propagation by applying the CIP method to these equations.²⁻⁴⁾

Figure 1 shows the aspect of the subgrid technique for the type-M CIP method. Here, Δx and Δy are course grid sizes, while Δx_s and Δy_s are subgrid sizes, respectively. The difference between the type-M and type-C CIP methods is the handling of a second-order special derivative (see ref. 11).

Figure 2 shows the treatment of the boundary course grid and the subgrid in the $\pm x$ -direction propagation for the type-M CIP method. In the first step, p and $\partial_y p$ are interpolated in the $\pm y$ -direction by third-order Hermite interpolation, whereas v_x , $\partial_x p$, and $\partial_x v_x$ are interpolated by first-order Lagrange interpolation. Although the procedures of the subgrid techniques for the type-M and type-C CIP methods are basically similar,¹¹⁾ the type-M CIP method employs the Lagrange interpolation.¹²⁾ Next, we calculate advection equations in the $\pm x$ -direction in the second step. Note that the sub grid technique for CIP analysis just needs to change the interpolating function in the sub grid region, because the CIP scheme is based on a two-point stencil's MOC.

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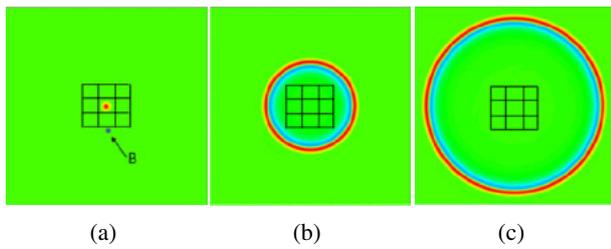


Fig. 3. (Color online) Sound pressure distribution obtained by type-M CIP analysis with subgrids. (a) $t = 10\Delta t$, (b) $t = 500\Delta t$, and (c) $t = 1000\Delta t$.

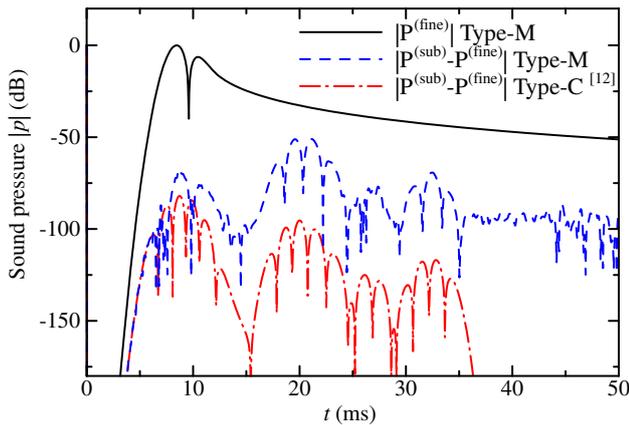


Fig. 4. (Color online) Absolute pressure value at point B in type-M and type-C CIP analyses: $|P_B^{(fine)}|$ and $|P_B^{(sub)} - P_B^{(fine)}|$.

We present numerical results obtained using the subgrid technique for type-M CIP analysis. The calculation parameters used are as follows: the direction of acoustic field propagation, $\pm x, y$ (two-dimensional analysis); course grid size, $\Delta x = \Delta y = 0.06\text{ m}$; subgrid size, $\Delta x_s = \Delta y_s = 0.02\text{ m}$; time step, $\Delta t = 3.657 \times 10^{-5}\text{ s}$; $\rho = 1.21\text{ kg/m}^3$ and $K = 1.4236 \times 10^5\text{ Pa}$.

Figure 3 shows the sound pressure distribution obtained by type-M CIP analysis with subgrids at $t = 10\Delta t$, $t = 500\Delta t$, and $t = 1000\Delta t$. The input pressure is driven from inside of the subgrids. Here, the meshed area is the subgrid region. We can ascertain the propagation behavior including that in the subgrid region. Figure 4 evaluates the error using subgrids by means of comparison of the absolute pressure value at point B (see Fig. 3). We also show the numerical results obtained using the subgrid technique for type-C CIP analysis.¹¹⁾ Calculation parameters of both analyses are on equal terms. It is confirmed that the boundary in the subgrids has good permeability characteristics with low reflection. However, the numerical error of the type-M CIP method is larger than that of the type-C CIP method for acoustic simulation with a subgrid system. Moreover, the type-M CIP method has an error of approximately -50 dB at around 20 ms. This is due to the reflection from both side boundaries between subgrids and course grids. The results indicate that the oblique reflection from subgrid boundaries must be carefully treated in the type-M analysis.

We also investigated the calculation time required for some subgrid models. Here, we used a PC with Intel Core i7-980X Extreme Edition 3.33 GHz. This processor has 6 cores

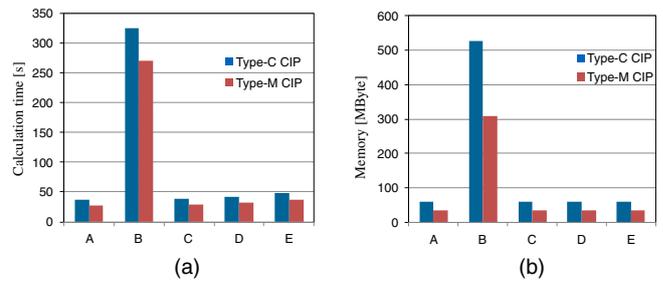


Fig. 5. (Color online) (a) Comparison of the calculation times. (b) Comparison of the results of memory use [calculation parameters (see Table I)].

Table I. Calculation parameters.

	A	B	C	D	E
Course grid: (0.06 m) ²	800 × 800	—	800 × 800	800 × 800	800 × 800
Fine grid: (0.02 m) ²	—	2400 × 2400	50 × 50	100 × 100	150 × 150

and 12 hyperthreaded cores, or effectively scales 12 threads. For all analyses, parallel computation using OpenMP was applied.

Figure 5(a) shows a comparison of the calculation times, where calculations are divided into 500 time steps. Figure 5(b) shows results of the memory use in CIP analyses. The subgrid model requires less calculation time and uses less memory than the fine grid model. The calculation time of the type-M CIP method is about 0.84 times smaller than that of the type-C CIP method, whereas the memory use of the type-M CIP method is about 0.58 times smaller. This is because the type-M CIP method is required to calculate finite differences instead of employing a second-order special derivative.⁴⁾ Here, Table I presents calculation parameters.

Using type-C and type-M CIP-MOC methods, we assessed a subgrid technique for the numerical simulation of sound wave propagation. The numerical results obtained by the type-C and type-M CIP methods with subgrid techniques were compared for a two-dimensional acoustic field. Examination results reveal that the correct treatment of the boundary between course grids and subgrids causes little reflection. The use of a suitable multisize grid reduces the time and memory necessary for calculation.

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